

Rise and Fall of Online Game Groups: Common Findings on Two Different Games

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ABSTRACT

Among many types of online games, Massively Multiplayer Online Role Playing Games (MMORPGs) provide players with the most realistic gaming experience inspired by the real, offline world. In particular, much stress is put upon socializing and collaboration with others as a condition for one's success, just as in real life. An advantage of studying MMORPGs is that since all actions are recorded, we can observe phenomena that are hard to observe in real life. For instance, we could observe how the all-important collaboration between people come into being, evolve, and eventually die out from the data to gain valuable insights to the group dynamics. In this paper, we analyzed the successes and failures of the online game groups in two different MMORPG, ArcheAge of XLGames, Inc. and Aion of NCsoft, Inc.. We find that there exist factors that influence the dynamics of group growth common to the games regardless of the games' maturity.

Categories and Subject Descriptors

J.4 [Computer Applications]: Social and Behavioral Sciences

General Terms

Measurement, Human Factors

Keywords

Online game community, Group dynamics, Massively Multiplayer Online Role Playing Game (MMORPG).

1. INTRODUCTION

1.1 Users' Behavior Aspect to Pursuit Profits

Massively Multiplayer Online Role Playing Games are one of the most popular forms of online gaming, with millions of dedicated fans throughout the world [1]. There are many online social networks (e.g. Facebook, Twitter) on the Internet. They have become the most comprehensive and useful human behavior data analysis platforms; they, however, are limited in terms of what we can observe from the full set of complex human behaviors.

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Among many online social platforms, online games, especially MMORPGs are the most similar to the real world. They provide a variety of actions (befriending, treasure hunts, item trading, and community joining) for the players to enjoy [2]. By analyzing these action logs, we can observe multi-relational interactions between humans. To this end, we focus on analyzing a group (usually called as a community or guild in game) in aspect of multi-relational interactions in online game virtual world.

In many games socializing and collaboration play an important role in the success inside a game, which is not at all different from how the real world works. Between users, they continuously influence each other by doing altruistic behaviors [3, 4] or malicious behaviors [5, 6]. These positive or negative influences affect the user's loyalty on the games. Like the personal relation, a user gets an influence from the belonged group. Furthermore, in real world or virtual world, we are all likely to become a member of various communities that define various aspects of our daily life, and we soon realize that many great achievements necessitate well-executed collaborations.

For this reason, the rise and fall of social groups have long been the focus of social science research. In our previous work [7], we found that in the early stages of a game (ArcheAge)'s adoption the prospect for benefits to a user is more important for a group's growth than any sense of loyalty. Based on our observations, these following findings are the most significant factors to affect a guild's birth and growth.

Findings 1: People tend to join guilds when they expect that tangible benefits (game money or valuable items). To gain these money and items, they collaborate intensively by doing castle sieges or raid quest between users.

Findings 2: Users switch guild membership to gain more profit (that is, strong guild is getting stronger, weak guild is getting weaker).

Based on these findings, we concluded that users firstly consider personal profit than social loyalty. Finally we concluded the social actions that can lead to an increase in an individual's profit are one of the significant factors that affect a guild's rise and fall (strengthening and expanding the lifetime of the guilds).

However, ArcheAge is in the early stage of online virtual community not mature, therefore, to see if any difference exists

between a game in its early stage and another that is considered mature, we encompass our analysis toward Aion, a more mature MMORPG, in a similar fashion and present the findings here.

1.2 Contributions

The contributions of this work are as follows.

- We performed the comparative analysis in two different games, ArcheAge and Aion. They are in the same genre (MMORPG) but the maturity is different. ArcheAge is in the beginning stage, and Aion is in mature stage. As far as we know, this research is the first one to perform comparative analysis between major MMORPGs by analyzing big dataset.
- This research can be used for a game system design to enforce user's feeling of belongingness in the virtual society. Finally, that can affect the guild's longevity and user's satisfaction positively.

2. RELATED WORK

We broadly classify current research activity on social groups into four categories, as shown in Table 1.

Table 1. Previous researches on group dynamics

Category	Dataset Used	Focus
Individual and social action [8], [9]	World of Warcraft	Personal history, Social features
	EverQuest II	Player engagement, Social influence
Social Action [10], [11]	MMOG forums	Community position, Community trust, Social value, MMOG continuance intention, Game knowledge, Membership duration, Average play time
	Aion	Group affiliation, Group dynamics, Group network, Implications
Group Structure [12], [13], [14]	Ning communities	Growth, Connectivity, Structural
	World of Warcraft	Guild structure, Guild organization & survival
	DBLP, IMDB, BLOGS, WIKI	Density, Intersection, Size, Growth, Core
Group Lifecycle [15], [16], [17]	Article Mobile phone call	Life time, Stationary, Phone-call, Co-authorship
	Email, Portal	Continuing, Shrinking, Growing, Splitting, Merging, Dissolving, Forming
	Online community publications	Inception, Creation, Growth, Maturity, Termination

Researches belonging to the Individual and Social Action category used the characteristics of the individuals and the group. Patil et al. [8] attempted to predict the membership of an individual in a group using the characteristics of the individual and the group, game activity patterns, and the social networks of

users from the World of Warcraft data. They found that that past loyalty, group stability, and social engagement were the most relevant factors for predicting the probability of quitting a guild. Kawale et al. [9] studied the phenomenon of churning in EverQuest II. They proposed a churn prediction model based on their examination of social influence among players and their levels of personal engagement in the game.

The researches in Social Action category used the degree of trust in the community and social interactions. Hsiao and Chiou [10] investigated how players' positions in an online game group affected their intention to continue playing an MMORPG. They provided support for the moderating effects of gaming knowledge and community size on the proposed model. Due to measure the impact of social capital, they selected seven features (i.e., community position, community trust, social value, intentions to continue the MMOG, knowledge of the game, membership duration, and average play time). Chung et al. [11] analyzed the characteristics of Aion groups from a socio-economic viewpoint. They found that growing groups exhibit more cohesive social interactions and balanced communication patterns.

The researches in Group Structure category dealt with structural features such as the size and density of the group. Kairam et al. [12] studied group growth via diffusion and non-diffusion during a group's formative stages from the perspective of individuals and groups. They reported that past growth features predicted the short-term growth more accurately, whereas structural features of the network predicted the long-term growth for smaller groups. They demonstrated that group clustering encouraged diffusion growth, and groups that grew more via diffusion tended to reach a smaller size. Ducheneaut et al. [13] found factors that described the success or failure of a gaming guild using data collected from the World of Warcraft. They claimed that expanding a guild while partitioning the members into small, balanced subgroups (in terms of class and levels) was good for accomplishing missions and clearing dungeons. Goldberg et al. [14] proposed an algorithmic framework for studying the evolution of communities in social networks. They identified that the lifespan of a community relates to structural parameters that were correlated with the early evolution of the community. They showed that density, intersection, and core size were significantly related with the lifespan of a group.

Lastly, the researches in Group Lifecycle category studied the birth, growth, decline, and demise of the groups. Palla et al. [15] identified the statistical properties of evolving groups through time. They formulated their analysis based on outstanding events in the evolution of a mobile phone network group, including growth, contraction, merger, splitting, creation, and destruction of the group. They demonstrated that large groups persisted longer if they were capable of dynamically altering their membership, while small and stationary community generally underwent minor changes but survived longer. Brodka et al. [16] attempted to predict the pattern of group evolution in e-mail communication and portal communities. Using a simple sequence of several preceding groups' sizes and events as an input for the classifier, they showed that decision trees and random forest were often the most accurate classifiers. Iriberry and Leroy [17] reviewed research on online communities, and organized the factors of success based on the lifecycles of information systems and the community type and genre. They proposed that different technology features are necessary depending on the developmental stage of the online community. They reviewed studies from such various fields as computer science, sociology,

management science, psychology, information systems, etc.. They presented the benefits of an online community to the individuals therein. They also classified the factors of success for each stage in the lifecycle of online systems (Inception, Creation, Growth, Maturity, and Death).

3. MATERIALS AND METHODS

3.1 Group Dynamics

In our previous work [7], we found that gamers are motivated more by benefit than by loyalty in choosing guild and community memberships in an MMORPG ArcheAge. As a result, game actions that can benefit the users become significant for the group's growth. We applied the same methodology to user behaviors observed in a mature game, Aion. The basic statistics of the two games and data are given in Table 2. ArcheAge's beta test was launched in South Korea on December 8, 2011; Aion was released in South Korea on November 25, 2008.

Table 2. Basic statistics of ArcheAge and Aion

	ArcheAge dataset	Aion dataset
Observational period	December 8, 2011 ~ February 25, 2012 (About 80 days)	April 10, 2010 ~ June 28, 2010 (About 80 days)
# of users	20,253	85,891
# of guilds	224	1,680
Dataset Age	Release: December 8, 2011 (+80 days from the service launch)	Release: November 25, 2008 (+580 days from the service launch)

3.2 Group Dynamics

The ArcheAge dataset contains all in-game logs for the period from December 8th of 2011 to February 25th of 2012, which covers the entire period of the 4th Closed Beta Test (CBT); this makes the game world only 80 days old, allowing us to observe the group dynamics in a very young, yet-to-mature game. During this period, 224 guilds were formed, and of the 20,253 unique users who played the game 6,309 players joined a guild. The Aion dataset contains all of the in-game logs for the period from April 10th of 2010 to June 28th of 2010. This game was launched on November 2008, making it more mature than ArcheAge. During this period, 1680 guilds were formed, and of the 85,891 unique users who played the game, 7,558 players joined a guild.

Fig. 1 shows the CDFs for the guild sizes found in ArcheAge and Aion. X-axis represents the guild size (measured by its maximum number of membership in an observational period), and Y-axis represents the cumulative rate of guilds for the guild size. The differences likely stem from their conditions for guild creation and expansion. ArcheAge guilds comprise five members at a minimum, with no upper bounds (as currently as 4th CBT test). Aion guilds comprise one member at a minimum, and has a level designation that limits the member numbers per level (30, 60, 90, 120, 150 members for guild level 1, 2, 3, 4, and 5, respectively).

In ArcheAge, about 14% of the guilds have more than 100 members, and the largest guild has 337 members. In Aion, about 14% of the guilds have more than eight members, and the largest guild has 66 members. To create a guild, users just pay a creation fee to the guild administration office located in each race's capital. Aion guilds have level specification with varying benefits. A

guild's level can increase when some conditions are met, e.g. the guild size, co-play frequency between guild members, and the amount of cyber assets collected. Guild members can also benefit (e.g., gain experience points) when the guild achieves a higher level. When users create a guild, users begin at rank 1, and the guild can have up to maximum 30 members. Upon reaching the size of ten members, users may choose to upgrade their guild to rank 2 for a fee. Rank 2 can have up to 60 members. By contrast, a guild in ArcheAge needs at least five players above level 5 to exist. A group of users can ask NPC guild managers to create a guild for them after paying some money. Guild members usually participate in hunting and siege wars together to gain more experience points. Some minor differences exist, although the purpose of a guild and main benefits are common between the two games.

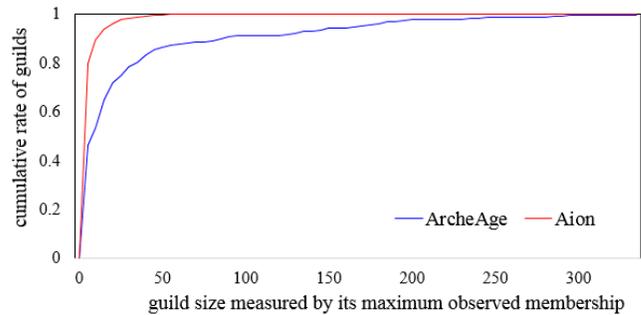


Figure 1. The membership size distribution of guilds in ArcheAge (blue) and Aion (red).

In general, the number of guild members is widely accepted as a success factor of guilds. In Fig. 2(a) and Fig. 2(b), we show how the increase and the decrease in guild membership are correlated with group activities in ArcheAge. Fig. 2(a) shows the group actions (party play [18] and siege [3] activities) per day by the users joining guilds.

In Fig. 2(a), X-axis represents the observational period by date. The colored section in Y-axis and left-most Y-axis value represent the count of various group activities (e.g. party creation, raid team creation, party join, and raid team join). The black colored curve and the right-most Y-axis value represent the cumulative number of members joined the group (guild). Many new members correspond to many group actions. Likewise, Fig. 2(b)'s X-axis and left-most value of Y-axis represent the same thing. The black colored curve and the right-most Y-axis value represent the cumulative number of members leaving the group. That is, many leaving members correspond to many group actions as well.

As shown in Fig. 2(a) and Fig. 2(b), when the profit sharing from the group action (such as party play and raid play) are not satisfactory, then the users leave the belonged guild to join another more beneficial guilds. Therefore, the user's satisfaction from the group activities can affect the rise and fall of the groups.

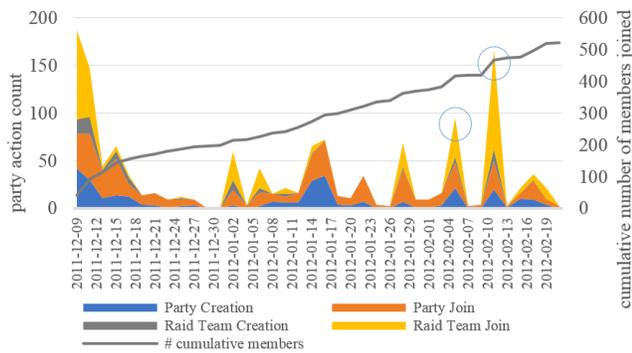


Figure 2. (a) The cumulative number of people joining and the concurrent group actions frequencies in ArcheAge.

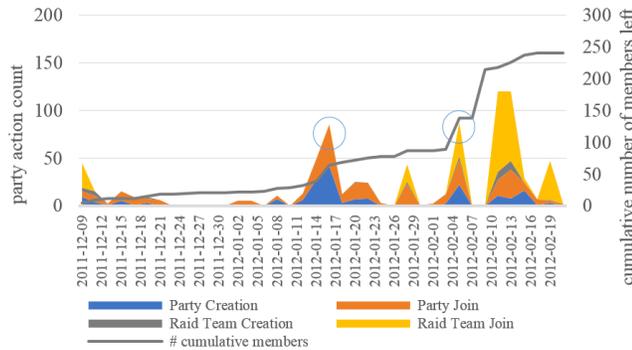


Figure 2. (b) The cumulative number of people leaving the guilds and the concurrent group actions frequencies in ArcheAge.

In Fig. 3(a) and Fig. 3(b), we also find the same aspects of the increase and the decrease in guild membership is highly correlated with group activities in Aion. There were big spikes of party play events when a there is a prominent change in the guild membership. For the first time, we expect that social tie will be getting stronger in a mature game. However, users show the same behaviors when joining and leaving the group activities regardless of the games' maturity (number of users, number of guilds, game's age and economic maturity). That means the rise and fall of the online game groups are highly related to the group activities, and users make a decision based on their profit than loyalty.

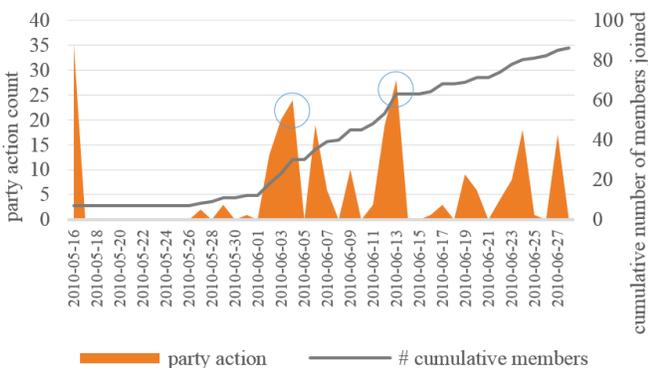


Figure 3. (a) The cumulative number of people joining and the concurrent group actions frequencies in Aion

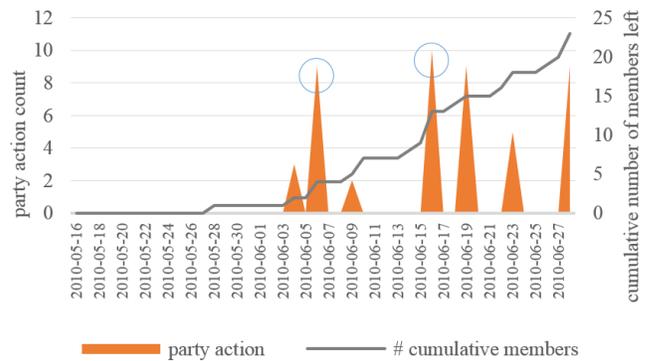


Figure 3. (b) The cumulative number of people leaving the guilds and the concurrent group actions frequencies in Aion

Fig. 4 shows the bar graphs of guild leave actions per user in ArcheAge and Aion. In ArcheAge, among 6,197 members 1,759 have left, and 464 of those have left guilds twice or more. In Aion the numbers are 7,758 and 2,059. 586 have left guilds twice or more. The ratios of the number of withdrawals are almost identical in both games.

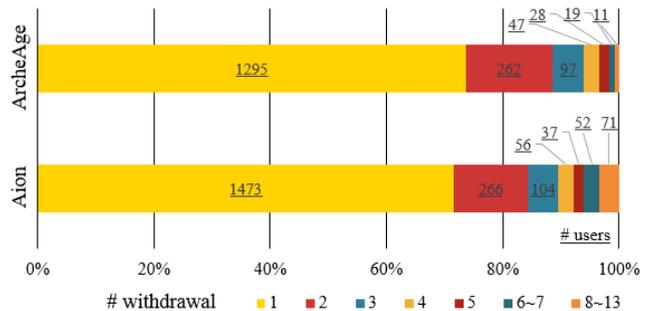


Figure 4. The frequency of withdrawal of the guild by players in ArcheAge (top bar) and Aion (bottom bar).

Fig. 5 shows the ratio of members who rejoin the same guild. In ArcheAge, 604 users out of 7,785 users rejoined a guild after withdrawal. About 30% of these users did rejoin activities twice or more. In Aion, 681 users out of 1,236 users rejoined a guild after withdrawal. About 45% of these users rejoined a guild twice or more.

In our previous work on ArcheAge, we found that subgroups moved in tandem from their original guild to an allied guild for castle sieges and moved back to their original guild after they are over. The same phenomenon is observed in Aion regardless of the game's maturity.

As the same, we firstly expect that Aion will be far from the frequent leaving and rejoining activities in the same guild (because that looks like shameful profit-seeking and loyalty-less behaviors). However, the users' behaviors are similar in overall.

The friendship network is the most fundamental and common social network in MMORPGs. Users can become friends by adding each other to their Friend Lists. A cross-analysis with the "join guild" action log enables us to see how the expansion of a guild correlates with the strength of the friendships between members. We conducted a study by analyzing two network transition phenomena, i.e., the guild-to-friend transition

(becoming friends after being affiliated with a common guild) and the friend-to-guild transition (becoming affiliated with a common guild after being friends).

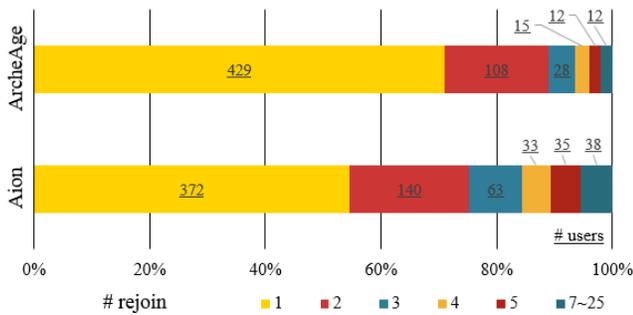


Figure 5. The frequency of joining of the same guild by players in ArcheAge (top bar) and Aion (bottom bar).

In Fig. 6 we show the order in which the users join guilds based on a sample study. Dark gray implies guild members becoming friends, while light gray implies friends joining a common guild. In ArcheAge, both are 40-60%, meaning the ordering is not significant. In Aion, we observe a noticeably higher ratio of friends joining the same guild in Aion. Other observations in this paper show similar results between the two games, but ArcheAge and Aion show differences in the following aspect: In ArcheAge, friend-to-guild transition is comparable to guild-to-friend transition, while in Aion, friend-to-guild transition is more frequent than guild-to-friend transition. This likely stems from the restriction on the number of users that can join a guild in Aion. This appears to be the reason why the friend-to-guild transition is more frequent than the guild-to-friend transitions.

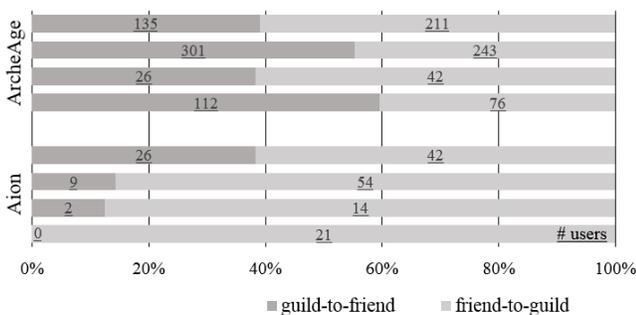


Figure 6. The ratio of the prevalence of network transitions (guild-to-friend, dark gray, versus friend-to-guild, light gray) in ArcheAge and Aion.

We also investigated whether there existed specific patterns in in-game chatting that correlated with the success of a guild. The results are shown in Fig. 7(a) and Fig. 7(b), where we show the frequencies of each chatting type in four different guilds. Starting from the top and moving down, the four bars show the results for a guild that is large and old, a guild that is small and older, a guild that is small and young, and a guild that is large and young. In Fig. 7(a), the membership sizes of the sample guilds in ArcheAge are 152, 14, 8, and 61, respectively. In Fig. 7(b), the membership sizes of sample guilds in Aion are 60, 10, 11, and 35, respectively.

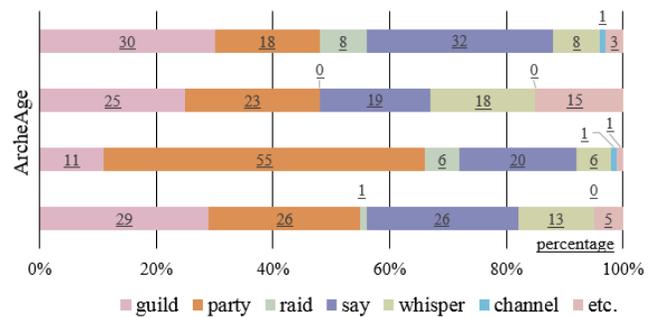


Figure 7. (a) The prevalence of distinct chat types in four representative guilds in ArcheAge.

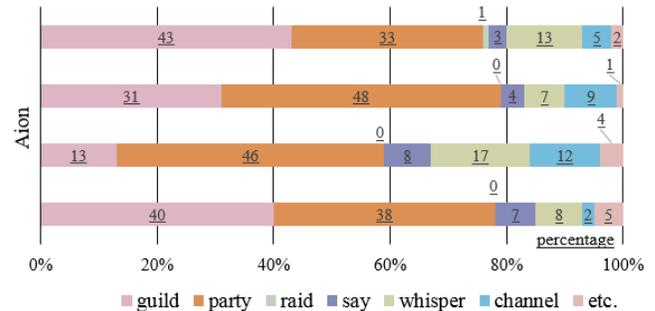


Figure 7. (b) The prevalence of distinct chat types in four representative guilds in Aion.

Fig. 7(a) and 7(b) commonly show the prevalence of chat types depending on the guild size and age.

The overall chatting type ratios are similar in both games. We see first that the guild that is small and young (3rd bar) shows a lower fraction of guild chatting, which is the dominant chatting type (about 30% in ArcheAge and about 40% in Aion) in the other three guilds. This fact demonstrates that the active communications occur during collective actions between guild members. In addition, these active communications are correlated strongly with the growth of the guild. We explore the overall trends found in four representative types of guilds in both games. However, we need to do a total inspection for all guilds as the future work to find another aspect not revealed yet.

4. Conclusions and Future Studies

In our previous work, we found in the early stages of the game prospect for benefit are the more important factor in the group's growth than personal or social loyalty. We applied the same methodology to the mature game and determined that ArcheAge and Aion yield very similar results, showing that people's motivations are consistent regardless of the maturity of their environments.

Our analysis also shows what users expect to gain from guilds. There are multiple advantages to belonging to a guild; In general, users can do a party play with guild members who tend to be more cooperative than a random partner would be. Moreover, users can spend their time efficiently, as it can take a significant amount of time to find people to play with. Users can also obtain valuable information, as they often share knowledge based on their game experiences. Since cooperation between players of differing levels in a guild is common, lower-levels can be more successful by playing inside the guild than alone, they can rise in level quickly, contributing to the success of the guild.

A group leader can sometimes affect a group's performance significantly, and we find that a group leader's competence and commitment are indeed important in online guilds.

Future research should focus more on the role of the social network in guilds. We plan to explore the changes in the friendship network, whisper network, trade network, and party-play network in MMORPGs to build a prediction model of the rise and fall of guilds.

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